Projection onto the Tangent Grid of a Parabola

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Abstract

This study gives the projection of geometric shapes (e.g., circles) onto the tangent grid of a parabola. The tangent grid is defined by the family of tangent lines to the parabola $y = \frac{1}{4}x^2$. I derive the transformation equations, explain the geometric interpretation, and provide visualizations of the projected shapes.

1 Mathematical Foundation

1.1 Parabola and Tangent Lines

Consider the parabola defined by:

$$y = \frac{1}{4}x^2.$$

The derivative of the parabola is:

$$\frac{dy}{dx} = \frac{x}{2}.$$

At a point t on the parabola, the tangent line is given by:

$$y = \frac{t}{2}x - \frac{t^2}{4}.$$

1.2 Tangent-Based Coordinate System

We define a coordinate system (t, v), where:

- t is the parameter for the tangent lines (equivalent to the x-coordinate of the point of tangency).
- v is the position along the tangent line.

The transformation from (t, v) to Cartesian coordinates (x, y) is:

$$\begin{cases} x = t + v, \\ y = \dfrac{t^2}{4} + \dfrac{tv}{2}. \end{cases}$$

1.3 Geometric Interpretation

The transformation maps points from the (t, v) space to the Cartesian plane, aligning them with the tangent grid of the parabola. This creates a warped coordinate system where geometric shapes (e.g., circles) are distorted based on the curvature of the parabola and the divergence of its tangent lines.

2 Projection of a Circle

A circle in (t, v) space is defined by:

$$(t - p)^2 + (v - q)^2 = r^2,$$

where (p, q) is the center and r is the radius.

Applying the transformation:

$$\begin{cases} x = t + v, \\ y = \dfrac{t^2}{4} + \dfrac{tv}{2}, \end{cases}$$

the circle is projected onto the tangent grid of the parabola. The resulting shape is a distorted version of the original circle, often resembling a cardioid or other interesting curves.

3 Visualization

Below is an example of a circle projected onto the tangent grid of the parabola. The original circle (in (t, v) space) is shown in purple, and the projected shape (in (x, y) space) is shown in navy.

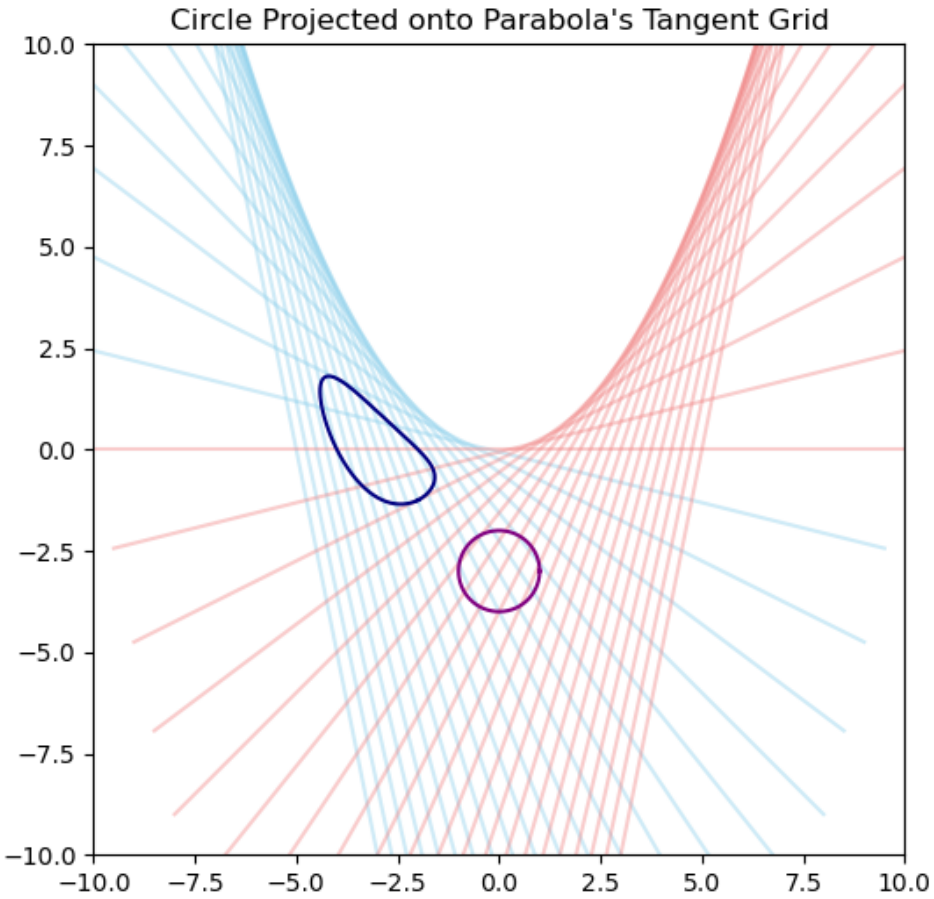

Figure 1: Projection of a circle onto the tangent grid of a parabola.

4 Applications

This projection has potential applications in:

- **Computer Graphics**: Creating visually interesting distortions of shapes.
- **Art and Design**: Generating aesthetic patterns using parametric transformations.

5 Conclusion

The projection of geometric shapes onto the tangent grid of a parabola is a fascinating application of differential geometry and parametric transformations.